Convective Radiation Fluid-Dynamics: Formation and Early Evolution of Ultra Low Mass Objects

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Abstract. The formation process of ultra low mass objects is some kind of extension of the star formation process. The physical changes towards lower mass are discussed by investigating the collapse of cloud cores that are modelled as Bonner-Ebert spheres. Their collapse is followed by solving the equations of fluid dynamics with radiation and a model of time-dependent convection that has been calibrated to the Sun. For a sequence of cloud-cores with 1 to 0.01 solar masses, evolutionary tracks and isochrones are shown in the mass-radius diagram, the Hertzsprung-Russel diagram and the effective temperature-surface gravity or Kiel diagram. The collapse and the early hydrostatic evolution to ages of few Ma are briefly discussed and compared to observations of objects in Upper Scorpius and the low mass components of GG Tau.

Key words: brown dwarfs, formation, planets, evolutionary tracks, isochrones

1. Models of young ultra low mass objects

Conventional models of ultra low mass objects, Burrows et al. (1993); Baraffe et al. (1998); Chabrier et al. (2000), are calculated using the equations, methods and assumptions of quasi-hydrostatic stellar evolution theory, Kippenhahn and Weigert (1990). Because they do not include fluid-dynamical effects and the treatment of radiative transfer and convection is simplified, the early dynamic and non-compact phases of stellar evolution can not be modelled. Consequently the evolutionary calculations cannot be started at physical states that are fairly long lived and accessible to observations — as molecular clouds and their substructures are — but they have to start at some intermediate state, that is largely dictated by technical issues.

Radiation-fluid dynamical calculations on the other hand, allow to calculate the collapse from observable cloud-states, but, as the static calculations do, have to assume spherical symmetry if it is desired to follow the evolution to the resulting, compact stellar states, that are observable at ages of typically a few Ma. 3D-effects, resulting from relaxing the symmetry-assumptions have been estimated by Wuchterl and Klessen (2001) by calculating the evolution including 3D isothermal cloud fragmentation for the solar case.

Fluid-dynamical models are able to connect clouds and stars. Since a couple of years that can be done with sufficient physical detail that is necessary for stellar models that can reproduce the Sun. The final theoretical evolutionary states (stars and brown-dwarfs after end of accretion) can now be sufficiently far evolved Wuchterl and Tscharnuter (2003) to approach ages that can be well observed, typically above a Ma. While the more general fluid-dynamical approach should be more realistic and may serve the general understanding of the formation processes, such models still have to make assumptions. So the question arises whether the classical, more handy approach is sufficient for all practical purposes as determining masses and ages of young objects.

Observational tests are needed to determine which approach is the more accurate and if the much simpler static models are fully sufficient for all practical tasks, as chronology and population-characterisation. In particular it is necessary to determine from which age on static tracks, that do not account for the formation process can be safely used. In particular the question has to be answered how long it takes until the thermal memory of the differences between the assumed fully convective static structures and the thermal signature of the collapse process, that results in partially convective structures at the end of accretion, are forgotten. The key, and still open question is if and when collapse structures converge to static, fully convective ones, i.e. onto a Hayashi-track. Two difficulties arise: (1) collapse calculations have to be advanced to ages at which high-quality observations are feasible, and (2) accretion effects have to have faded and the

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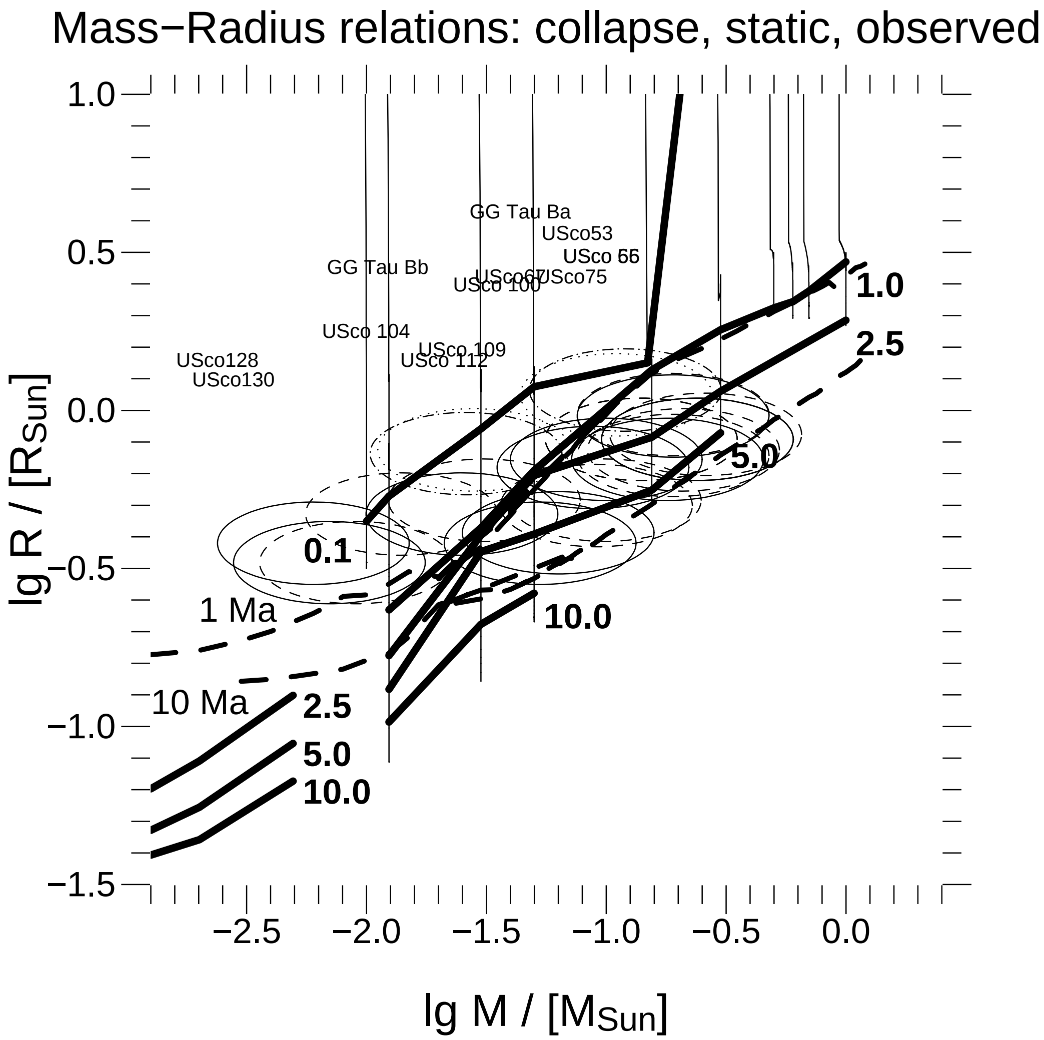

Fig. 1. The mass radius relation for young ultra low mass objects. Empirical determinations based on spectroscopic observations and analysis with classical model atmospheres, Mohanty et al. (2004a,b) are compared to hydrostatic, 'hot start' stellar evolution-like models *(dashed isochrones from 'Lyon' models,Baraffe et al. (1998); Chabrier et al. (2000))* and radiation fluid-dynamical calculations of cloud collapse. *Thin lines* are evolutionary tracks for optical depth radius $R_\tau(t)$ and optically thick mass M_τ from radiation-fluid-dynamical calculations of the collapse of marginally gravitationally unstable Bonnor-Ebert-spheres of various masses and thick lines the respective isochrones. Note that tracks for $M_\tau(t)$ for a given cloud mass M are not exactly vertical, especially towards the higher masses, because the objects are still accreting in the plotted range. M_τ is only becoming constant towards the end of the tracks as accretion processes fade. Isochrones are labelled with age in Ma. Error ellipses for spectroscopic measurements of very young very low mass objects are overplotted. *Dashed and full ellipses, for U Sco* and *dotted and dash-dotted ellipses, for GG Tau* refer to the two methods applied by Mohanty et al. (2004a,b) to determine the radii from measured fluxes, using two different infrared bands with the spectroscopically derived effective temperatures. Object identifiers are shifted to the upper left, for clarity. Isochrones in the lower left are for planets formed by core-accretion and envelope capture, in the framework of the nucleated instability hypothesis.

photospheres also need to be essentially static. Clearly static structure models cannot account for too early, non-static evolutionary phases. In practice also static stellar atmosphere models are used in quantitative spectroscopy of young stars. Hence essentially non-accreting objects are a precondition for their accurate application. Therefore a comparison cannot be performed in a clear way until the real objects are in an essentially static regime: they must be sufficiently 'mature', i.e. near their final mass. To carry out a comparison at ultra low mass objects has to key advantages: (1) low mass objects are more mature, i.e. closer to their final mass when they appear, cf. Wuchterl and Tscharnuter (2003), and (2) fluid-dynamical calculations can be advanced further in that mass range. The main cause is likely related to the smaller compactness, $R_\tau/H_P = r_{\rm acc}/R_\tau = GM/(R_\tau c_{\rm T}^2)$ of young low mass objects, resulting in less demanding numerical resolution requirements, cf. Wuchterl and Tscharnuter (2003). $r_{\rm acc}$ is the accretion radius and $c_{\rm T}$, the isothermal sound-speed.

These theoretical advantages are confronted to the observational challenges stemming from the inherent faintness of low-mass objects and the difficulties involved in calculating theoretical spectra for their relatively low gravity, ultra cool atmospheres. But recently Mohanty et al. (2004a,b) have presented a spectroscopic study of cool objects in the Upper Scorpius (U Sco) region and the low mass companions in the GG Tau system.

2. Equations and initial conditions

The equations solved here are the non-linear, time-dependent fluid-dynamical equations for radiating fluids with energy transfer by radiation and convection as described in Wuchterl and Tscharnuter (2003). The time-dependent convection model is very similar to mixing length theory in a local static limit. It is calibrated to the solar convection zone and tested by RR-Lyrae pulsations (cf.Wuchterl and Feuchtinger (1998)). Calculations start with a marginally gravitationally unstable, isothermal Bonnor-Ebert-sphere of given mass. Object properties are obtained by numerically determining the time evolution that follows from solving the model-equations in a constant volume for the initially hydrostatic and isothermal sphere. Calculations are continued until progress in dynamical model-time becomes inefficient as a function of real time, which typically occurs after 3 days of computing on a GHz processor, and the calculation of a few 10^5 time-levels.

3. Collapse of Bonnor-Ebert spheres

Since the situation is much more general than for conventional hydrostatic calculations, I briefly summarise how object properties are obtained. For a given cloud mass defined by the initial state, the solutions of the evolutionary equations, describe the flow of matter and radiation at any moment t. From the solution at t, the optical depth radius R_τ is determined by integrating the Rosseland-mean optical depth, τ inward from the outer boundary of the calculation, i.e. the original outer radius of the parent cloud, to the point where it reaches $\tau = 2/3$ (cf. Wuchterl and Tscharnuter (2003)). In this way we obtain a photospheric radius, R_τ. The effective temperature is determined from its definition and the luminosity using $R = R_\tau$. Masses shown are the optically thick masses, M_τ, i.e. the mass that is interior to R_τ. Note that M_τ is a result of the calculation and dM_τ/dt is the (time-dependent) mass-accretion-rate of the object, that also results from the solution. For an overview we refer to the isopleths given by Wuchterl and Tscharnuter (2003).

3.1. Evolutionary tracks

For each initial, marginally unstable Bonnor-Ebert-sphere of mass M, evolutionary tracks give the total luminosity at the outer cloud boundary, L — resulting from nuclear energy input, contraction and including all accretion effects as veiling —, the effective temperature, as defined above, the optically thick mass, M_τ i.e. the mass inside the photosphere, as a function of time. A track is defined by cloud-mass M and carries variable M_τ. If the entire cloud fragment mass is accreted, $M_\tau = M$ after the end of significant accretion. That is the case towards the end of all tracks presented here.

3.2. Isochrones

Isochrones are obtained for the specified age-values by calculating the age as defined by Wuchterl and Tscharnuter (2003) along all the evolutionary tracks for the available initial cloud

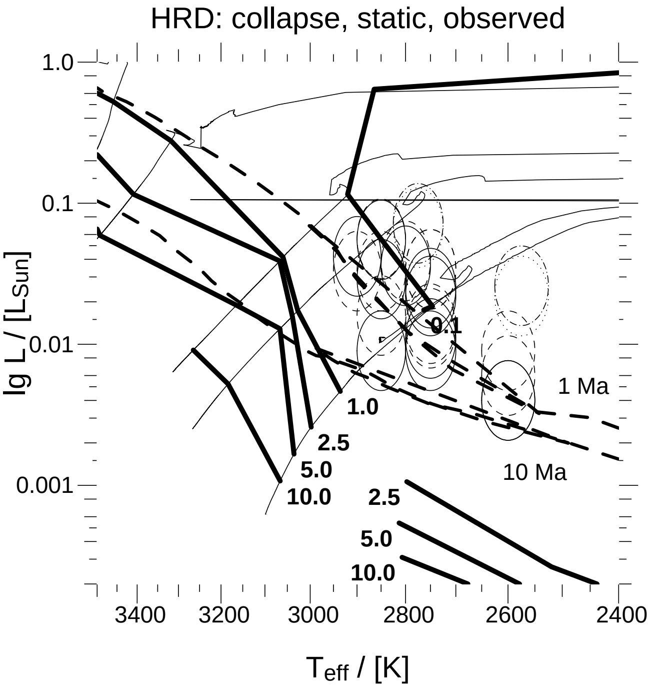

Fig. 2. Comparison of static and collapse calculations and observations relating to young ultra low mass objects in the theoretical Hertzsprung-Russel diagram. As in Fig. 1 dashed lines are isochrones from hydrostatic, hot start calculations, thick lines are isochrones and thin lines evolutionary tracks, both for the collapse of Bonnor-Ebert-spheres. Tracks are for cloud masses of 0.03, 0.015, 0.005, 0.003,and $0.0013\,\mathrm{M}_\odot$, from left to right at 1 Ma. Isochrones are labelled with the respective ages in Ma. Error ellipses show the spectroscopic determinations by Mohanty et al. (2004a,b), dotted and dash-dotted ellipses refer to the GG Tau components. Note that isochrones from both sets of 'Lyon' models, are plotted where the mass-ranges of the two studies overlap. That leads to two nearby dashed lines for mid-mass/mid-temperature ranges.

masses and connecting the points on the evolutionary tracks, that correspond to the age-value, with straight lines. The age defined by Wuchterl and Tscharnuter is basically the time elapsed since the first moment when an effective temperature can be defined. So ages are measured from the moment of formation of a photosphere. The photosphere separates the flow into an interior optically thick part (the later star or brown dwarf) and an optically thin outer part. The latter originally corresponds to the outer protostellar envelope and later the familiar classical stellar photosphere[1]. Photosphere formation coincides with the formation of the first or outer protostellar cores and, for a solar mass cloud, occurs within 10 ka after the end of the isothermal phase. Consequently time spent in the isothermal phase, during parent cloud fragmentation and near to hydrostatic cloud phases is not included in the age. However, time spent during accretion, after an optically thick, hydrostatic embryo has formed, is included. As

[1] The compactness mentioned above is the ratio between the scale height in the photosphere and the radius of that photosphere. All structures discussed here do have $\lg \mathrm{compactness} > 2$ at ages exceeding 1 Ma and hence comparison to classical, plane parallel photospheres, that do not account for sphericity effects should be geometrically accurate.

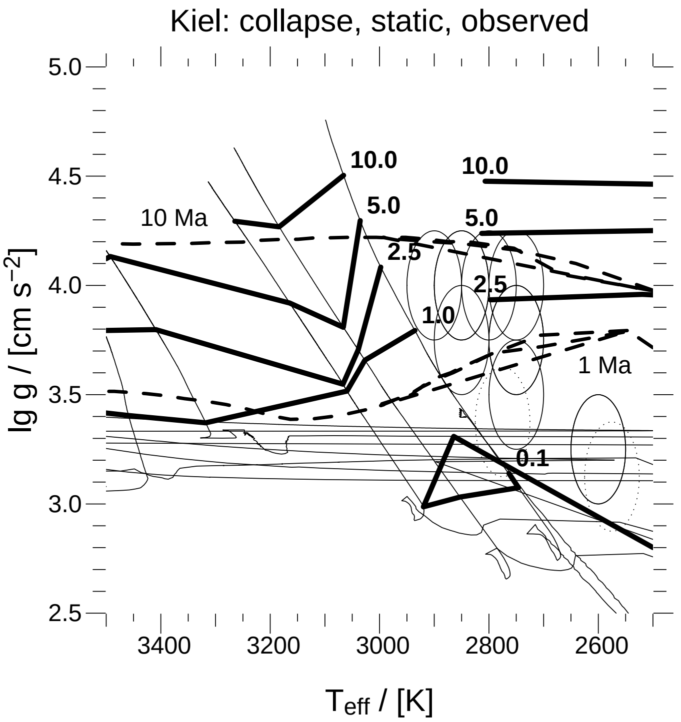

Fig. 3. Comparison of hydrostatic, hot-start and collapse calculations to spectroscopic observations in the Kiel diagram. Same as Figs. 1 and 2, see notes there, but now lg of the surface gravity in cgs-units, determined at R_τ is plotted versus effective temperature. Ages are labelled in Ma. Note the none-uniqueness at low gravities, the curly shape of the 0.1 Ma isochrone and the apparent jump of $\lg g$ across the gap between "stellar-like" objects formed from the spherical collapse and the "planet-like" objects that formed via core growth and envelope capture in the framework of the nucleated instability hypothesis of giant planet formation. Note that $\lg g$ increases upwards for easier comparison with Mohanty et al. (2004a). Dotted ellipses refer to the GG Tau components.

a rule of thumb, that amounts to about three to four initial cloud free-fall times, cf. Wuchterl and Tscharnuter (2003). Because not all tracks for all cloud-masses reach all specified ages, some isochrones are shorter then others. Typically lower mass clouds can be followed to larger Wuchterl and Tscharnuter ages but there is an exception around the opacity limit of a few Jupiter masses where such defined ages presently are restricted to below 1 Ma. Consequently there is a gap between the lower mass end of isochrones resulting from Bonnor-Ebert collapse and the upper end of planetary isochrones, resulting from nucleated-instability calculations, that are shown for reference.

4. Discussion and conclusion

For a discussion of the spectroscopy of Upper Sco objects and the low mass components of GG Tau, Mohanty et al. (2004a,b) see the contributions of Basri, Mohanty and Reiners, resp., in this volume. Since absolute ages presently cannot be determined observationally we check for coevality by plotting isochrones. Assuming coevality according to the respective theoretical age concepts, observed objects should scatter around isochrones corresponding to the respective ages of the GG Tau system and the Upper Scorpius region. Note that the age concepts of hydrostatic and collapse calculations differ fundamentally. The stellar-evolution-style quasi-hydrostatic calculations count ages essentially from their assumed initial state, whereas ages for the collapse calculations are counted from a physically well-defined zero age that is linked to the first formation of a photosphere, see Wuchterl and Tscharnuter (2003), for discussion.

Theoretically an interesting and somewhat surprising fact is the close agreement of the collapse and hydrostatic, hot-start calculations, at 1 Ma, for masses between 0.8 and 0.03 solar masses, in the M-R-plot, Fig.1. This may serve as a common age reference for the two approaches. Note that Wuchterl and Tscharnuter (2003) use a mixing-length parameter $\alpha_{\mathrm{ML}} = 1.5$, that is calibrated to the Sun (Wuchterl and Feuchtinger (1998)) and accurately reproduces the solar convection zone, whereas the 'Lyon' models, available in the required mass, range use $\alpha_{\mathrm{ML}} = 1.0$ that fails to do that. The structure-differences stand out clearly in the later isochrones in Fig. 1: the collapse models show a convergence of isochrones between 0.05 and 0.02 solar masses due to D-burning slowing down the contraction, whereas contraction is faster then in the static models for higher masses because D has been burnt during the embedded phases. Collapse-isochrones are consistent with ages of 1 to 5 Ma in Fig. 1, except for the two lowest mass U Sco objects that are best described by 0.1 Ma Wuchterl-Tscharnuter ages, and the GG Tau components (0.1 to 2.5 Ma). In the HRD and Kiel diagrams, Figs. 2,3, the observations are somewhat off the tracks, but there is no space for discussion here (see Basri, this vol.). Given the fact that all parameters are kept fixed after calibrating the equations to the Sun, the results of Bonnor-Ebert collapse are close to the observational evidence for fundamental parameters of ultra low mass objects. Empirically the Wuchterl-Tscharnuter ages of 0.1 Ma obtained from the models for the two lowest mass USco objects are unrealistically too young. More work is needed to reconcile the models and the observations.

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