Convection and giant planet formation

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Abstract.
Convection is of key importance in stellar structure, evolution and formation. Only recently the importance of a detailed treatment of convection has been discussed in the context of giant planet structure. I will show that 1. Mixing length convection significantly reduces the critical mass for the onset of efficient gas accretion and the formation of massive giant planet envelopes, 2. Current, increased molecular opacities point in the same direction, so that it may be concluded that less efficient transfer processes result in smaller critical masses, 3. The spreading of convection in the envelopes of proto giant planets leads to a substantial increase in the envelope mass at roughly constant core mass, 4. Time-dependent convection plays a key-role in damping nonlinear hydrodynamic waves at the critical mass and the onset of hydrodynamic accretion of large envelopes. The discussion will be based on new numerical calculations of the radiation- hydrodynamics of the envelopes of proto-giant planets, using a time-dependent theory of convection that reduces to mixing-length theory in the static local limit.

1. The Core-Nucelated Instability Hypothesis

Giant planet formation is discussed here in the context of the core-nucleated instability hypothesis, i.e. as the consequence of the formation and growth of the solid building blocks of rocky solar system objects as terrestrial planets, moons and asteroids. According to the 'planetesimal hypothesis' planets grow within circumstellar disks via pairwise accretion of km-sized solid bodies, the so-called planetesimals. Sufficiently massive planetesimals embedded in a gravitationally stable preplanetary nebula can capture large amounts of gas and become the cores of giant planets (e.g. Wuchterl 1995a, Tajima and Nakagawa 1997). But how much gas at what core mass? Presently the envelope-mass for a given core is usually calculated using zero entropy gradient convection. That means that convection is assumed to be very efficient and radiative losses from eddies are neglected. The temperature gradient in convectively unstable regions is approximated by the adiabatic gradient (e.g. Mizuno 1980, Pollack et. al. 1996, Wuchterl 1993). I will focus here on how the envelope masses change when more recent opacities are included and mixing length theory is used to describe convective energy transfer in the envelopes of proto giant planets.

The onset of formation of massive envelopes is characterized by the so-called critical and crossover masses. Typical values for the critical mass (largest static envelope for growing core) are in the range of 10 to 15 earth masses, for standard (minimum mass) assumptions about the preplanetary nebula. Pollack et. al. (1996) obtain crossover masses ($M_{\mathrm{high\ Z}} = M_{\mathrm{low\ Z}}$) in the range of 10 to 30 $\mathrm{M}_{\oplus}$ for various assumptions about planetesimal accretion.

2. The Critical Mass and Energy Transfer

I present envelope-mass calculations that are based on a new, time-dependet convection model (Wuchterl and Feuchtinger 1998) that is derived from the model of Kuhfuß (1987). It closely approximates standard mixing length theory in a local, static limit and has been succesfully tested in calculations of the Sun and RR-Lyrae pulsations. To determine the critical mass, the protoplanetary envelopes are assumed to be in hydrostatic and thermal equilibrium. The static structure equations are solved with a protoplanetary version of the VIP-code (Dorfi and Feuchtinger 1995) where convection is added as outlined by Wuchterl (1995b). The extended Kuhfuß model is used with the standard parameters given by Wuchterl and Feuchtinger (1998). For the formulation of

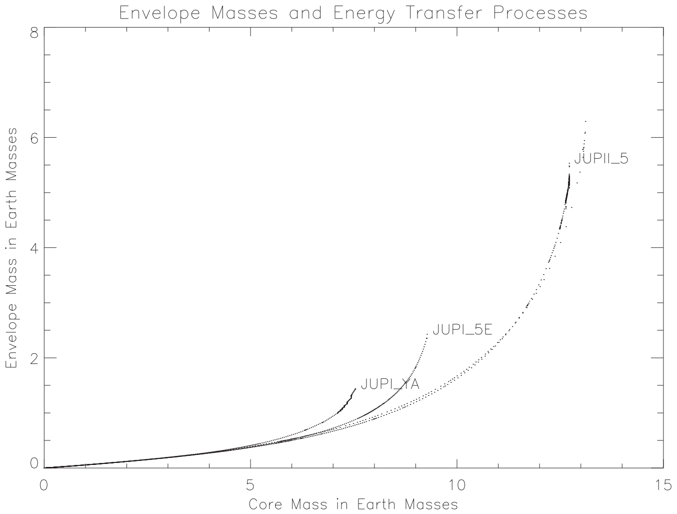

Figure 1. Envelope masses of proto giant planets and energy transfer processes. The masses of static protoplanetary envelopes are plotted as a function of the core mass for a standard 'Jupiter' case. The two curves labelled JUPII_5 are a result from Wuchterl (1993) and a recalculation with the planetary VIP-code, respectively, both for zero entropy gradient convection. JUPI_5E is the same, but for mixing length convection, and JUPI_YA for mixing length theory and updated molecular opacities.

equations and boundary conditions see Wuchterl (1991a). Calculations are made for Mizuno's (1980), minimum mass Jupiter conditions and constant core accretion rate: $10^{-6}\mathrm{M}_{\oplus}/\mathrm{yr}$. Those assumptions are identical to the 'Jupiter' cases of Wuchterl (1993,1995a). The new calculations, however are made for three cases with different treatment of energy transfer: JUPII_5: zero entropy gradient convection, JUPI_5E: mixing length convection, JUPI_YA: mixing length convection and 'Alexander' molecular opacities (Alexander and Ferguson 1994), in a compilation by Götz (1993). Results are shown in Fig. 1. Mixing length convection reduces the critical mass relative to the value for zero entropy convection. The updated (higher) molecular opacities result in a further reduction. Altogether the value of the critical core mass is reduced by roughly a factor of two relative to the value obtained for zero entropy gradient convection. Together the improvements of the energy transfer result in: (1) larger envelopes at given core mass, (2) smaller critical envelopes and (3) a reduction of the *critical core mass* from about 13 to 7.2 Earth-masses.

3. Envelope accretion at various orbital radii

The early phases of giant planet formation are almost static (see Wuchterl 1993) and the protoplanetary envelopes are well approximated by the static envelopes described in the last section. This changes when the envelopes become more massive and dominate the core. An interesting question is to ask for the core mass that is needed to initiate gas accretion to a few hundred earth masses at various orbital radii.

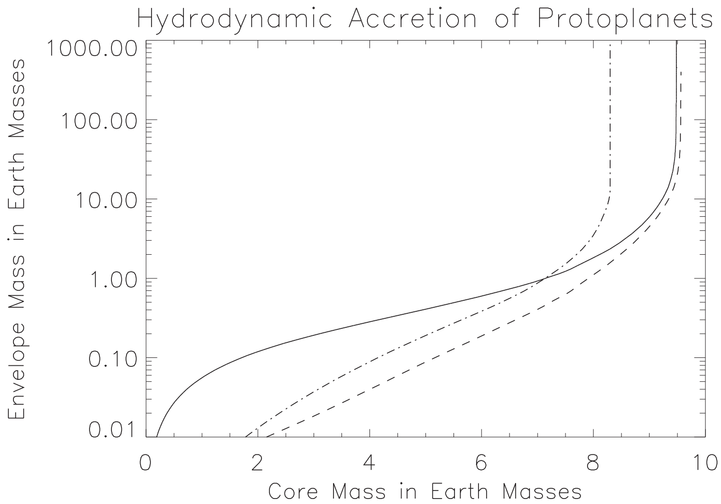

Figure 2. Evolution of proto giant planets with mixing lenght convection. Envelope masses as obtained from hydrodynamic accretion calculations are plotted as functions of core mass for various locations in a minimum mass nebula: 'Vulcan' (0.052 AU), dash-dot, 'Jupiter' (5.2 AU), full, and 'Neptune'(17.2 AU), dashed.

In Fig. 2 the results of radiation-hydrodynamical calculations of envelope accretion with the same set of equations as described in the last section but now fully time-dependent are shown for locations at 0.05 AU in the Hayashi et al. (1985) nebula and for Mizuno's (1980) 'Jupiter' and 'Neptune' cases. Orbital radii and nebula temperatures and densities for the three cases are 'Vulcan' (0.052 AU,1252 K, $5.3\,10^{-6}$ g/cm^3), 'Jupiter' (5.2 AU, 97 K, $1.5\,10^{-10}$ g/cm^3), and 'Neptune'(17.2 AU, 45 K, $3.0\,10^{-13}$ g/cm^3), respectively. The core accretion rate is 10^{-6} $M_\oplus$/yr, a standard value that is consistent with detailed studies of core accretion after the initial runaway phase (cf. Pollack et. al. 1996), that typically last one Myr. Hydrodynamic accretion to a few hundred earth masses is found for core masses of 8.30, 9.48 and 9.56 $M_\oplus$ respectively.

4. Conclusion

The application of mixing length theory, that correctly describes the solar convection zone and RR Lyrae pulsations, to giant planet formation results in lower critical core masses than obtained previously with simplified zero entropy gradient convection. The core masses that are necessary to initiate hydrodynamic gas accretion of giant planets are reduced to 8.30, 9.48 and 9.56 earth masses at 0.052, 5.2 and 17.2 AU respectively.

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