Gas accretion and giant planet formation

Günther Wuchterl

Institut für Astronomie der Universität Wien, Türkenschanzstraße 17, A–1180 Wien, Austria

Abstract The nucleated instability (or core accretion) hypothesis assumes that condensible element cores grow in the presence of the solar nebula and cause the formation of the massive gaseous envelopes of the giant planets. Once a core of a few earth masses of solids is accumulated a comparable amount of nebula-gas forms an equilibrium envelope around it. For a given core mass the properties of such core-envelope proto giant planets are largely independent of the distance from the star and the nebula conditions. The equilibrium envelopes are "stable" up to a so called critical or crossover mass. This mass is typically 20 earth masses, i.e. above the masses of Uranus and Neptune but below the masses of Jupiter and Saturn. If a proto giant planet arrives at the critical mass, the further accretional history is decided by the dynamical properties of the envelope and the timing due to solid body accretion, the pacemaker of the long-term evolution. Under conditions that typically occur at relatively high nebula densities gas accretion can overtake solid body accretion and the envelope can grow to large, i.e. Jupiter-like masses.

0.1 The Planetesimal Hypothesis

According to the 'planetesimal hypothesis' (see e.g. Lissauer 1993) planets grow within circumstellar disks via pairwise accretion of km-sized solid bodies, the so-called planetesimals. Sufficiently massive planetary bodies embedded in a gravitationally stable preplanetary nebula can capture large amounts of gas, thereby producing Jovian type planets (Mizuno 1980, Hayashi et al. 1985, Bodenheimer and Pollack 1986, Wuchterl 1991a,b, 1995, Pollack et al. 1996, Tajima and Nakagawa 1995, 1997).

I will discuss the process of nebula gas-accumulation by condensible element planetary embryos (terrestrial or icy) as studied by the construction of detailed models of protoplanetary envelopes. I refer to the chapters by Artymovicz and Conradini for a discussion of the effects of proto giant planets with given mass on the ambient nebula and the resulting processes of gap-opening and orbital migration.

0.2 How much nebula gas is bound to a planetary embryo?

The question of how much nebula gas is bound to a planetary embryo has been investigated with an increasingly detailed description of the envelope structure. In the following I will call such an embryo, that grows by impacting planetesimals, a ‘core’. Below a few earth masses the gaseous envelopes are negligible, consistent with the formation of terrestrial planets. At higher core-masses the resulting planetary masses depend on the assumptions made about the gaseous envelopes surrounding the cores. Two equilibria play a key role in the discussion of proto giant planets and their masses: (1) the assumption of hydrostatic equilibrium, a force-balance, where the combined gravity of the core and its gaseous envelope is balanced by pressure gradients; (2) the so-called ‘thermal equilibrium’, an energy-balance, where the energy liberated by the impacted planetesimals is balanced by heat losses to the ambient protoplanetary nebula.

If both equilibria are assumed, protoplanets can only grow to a maximum mass. Mizuno (1980) found this ‘critical mass’ to be about 20 earth masses ($M_\oplus$), independent of the location and conditions in the solar nebula. While this critical *core mass* of (typically 13 $M_\oplus$) is consistent with the condensible element masses inferred for the present giant planets, the total mass falls short of Jupiter's mass by more than one order of magnitude and is significantly larger than Uranus' and Neptune's. Above the critical core mass no static envelopes are known and hence one of the equilibria apparently does not hold.

Stevenson (1982) has given a simplifed analytical model for the envelopes of proto giant planets. He showed that the critical mass depends on the mean molecular weight, $\propto \mu^{-12/7}$, and weaker on the solid body accretion rate, $\propto \dot{M}_{\mathrm{core}}^{3/7}$ and the mean opacity $\propto \kappa^{3/7}$.

In Fig. 0.1 I show the effects of a detailed treatment of convection (standard mixing length theory) and updated molecular opacities (compiled by Götz 1993, based on Alexander opacities, cf. Alexander and Ferguson 1993) for a standard ‘Jupiter’-case (cf. Wuchterl 1993). Mixing length convection reduces the critical mass relative to zero entropy convection and the new (higher) molecular opacities result in a further reduction. Altogether the value of the critical core mass is reduced by roughly a factor of two. The improved treatment of the energy transfer processes therefore reconciles the critical core masses for proto giant planets, obtained for standard assumption about the protoplanetary nebula with the recent interior models of Jupiter and Saturn (Chabrier et al. 1992), that show smaller inferred core masses than previously.

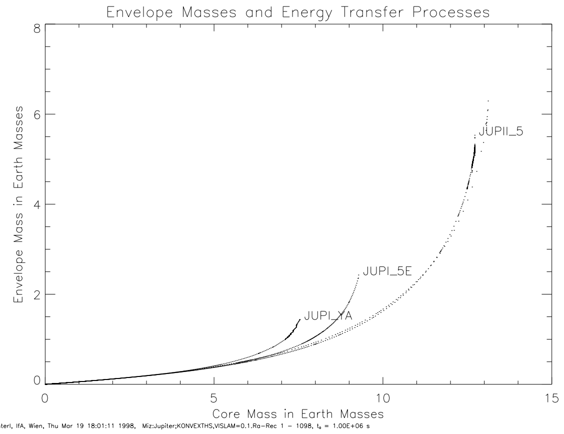

Figure 0.1: Envelope masses of proto giant planets and energy transfer processes. The masses of static protoplanetary envelopes are plotted as a function of the core mass for a standard 'Jupiter' case. The two curves labelled JUPII_5 are a result from Wuchterl (1993) and a recalculation with a new code, respectively, both for zero entropy convection. JUPI_5E is the same, but for standard mixing length convection, and JUPI_YA for standard mixing length theory and updated molecular opacities.

Quasi-hydrostatic studies (Bodenheimer and Pollack (1986), Lissauer et al. (1995), Pollack et al. (1996)) allowed the protoplanetary envelopes to depart from thermal equilibrium but assumed the hydrostatic equilibrium to hold. Quasi-hydrostatic envelopes may contract or expand slowly, i.e. they are not allowed to move at velocities higher than a small fraction of the sound speed. The forces almost balance under this assumption and the net-acceleration on the gas remains close to zero. The general result of those studies is that proto giant planets may grow to masses larger than the critical mass found for static proto giant planets. The accretion process is assumed to be terminated by limited supply from the solar nebula. Limiting processes may be nebula dissipation and/or tidal truncation of the gas flow from the nebula into the envelope, caused by the growing proto giant.

The most general, hydrodynamic studies concerning envelope accretion of proto giant planets (Wuchterl 1991a,b, 1995) do not assume energy- or force-balance. Here, the *motion* of the gas is calculated together with energy transfer, heating and cooling. Wuchterl (1995) found, that the evolution of a proto giant planet beyond the critical mass depends on the details of the envelope structure. Protoplanets with radiative outer envelopes eject parts of their envelopes at the critical mass and evolve to masses that are comparable to Uranus' and Neptune's. In contrary, protoplanets with convective outer envelopes continue to grow beyond the critical mass and accretion proceeds up to Jupiter's mass.

In summary: Below a few earth masses planetary embryos gravitationally attract only small amounts of nebula gas. As a consequence the gaseous envelopes have negligible masses. Overall they resemble terrestrial type planets. From a few earth masses to the critical value an increase in the core mass significantly increases the envelope mass. Core and envelope masses become comparable. Beyond the critical mass of about 15 earth masses the further growth of proto giant planets depends on the interior structure of the envelopes.

0.3 How do planetary masses depend on nebula conditions?

A key result of the study of giant planet formation was the insensitivity of protoplanetary structure to the conditions in the protoplanetary disk. Given the mass of a core and its growth rate, the mass of its envelope in hydrostatic and thermal equilibrium was found to be almost independent of the density and temperature in the solar nebula and the distance from the star (Mizuno 1980). Apart from the effects discussed in the last section, proto giant planets should have essentially the same mass irrespective of their location in the nebula.

The weak dependence of protoplanetary envelope structure on the nebula conditions in the vicinity of the critical mass is illustrated by the two lower curves in Fig. 0.2. Both are mixing length calculations for the 'Jupiter'-case of Wuchterl (1993) but the nebula density is reduced by more than 10 orders of magnitude for the lower curve.

The reason for this non-dependence of protoplanetary masses on the conditions in the ambient nebula is the radiative structure of the outer parts of the gaseous envelopes (Stevenson 1982). Wuchterl (1993) examined the convective stability of these radiative outer envelopes. According to his results, the outer envelopes become convective for nebula densities slightly higher than in standard minimum reconstitutive mass solar nebulae. Once the outer envelopes are convective, the masses of the proto giant planets start to *depend* on the nebula conditions. Wuchterl (1993) found envelopes from $< 6 - 48$ $M_\oplus$ for the same core mass but different nebula conditions. The critical mass for protoplanets with convective outer envelopes increased up to a total mass of $60\,M_\oplus$. Wuchterl's (1993) results are illustrated in Figs. 0.2,0.3 by new calculations using mixing length theory to describe convective energy transfer.

Even the largest critical 'convective' proto giant planets still fall short of Jupiter and Saturn, an indication, that gas-accretion has to continue beyond the critical mass and will involve quasi-hydrostatic and/or dynamical phases. Wuchterl (1995) hydrodynamically calculated the envelope accretion beyond the

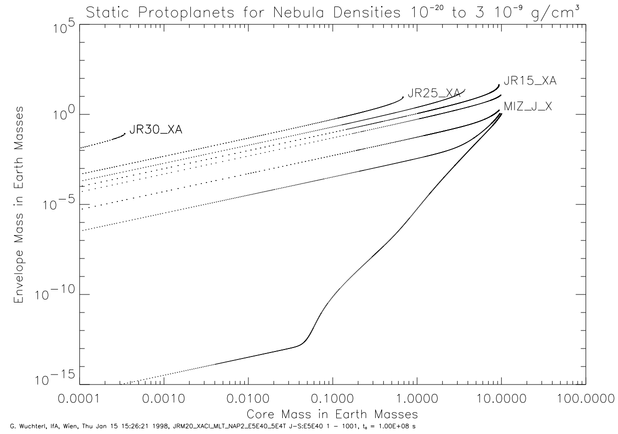

Figure 0.2: Masses of static protoplanetary envelopes as a function of core mass. All results are obtained for the 'Jupiter' case of Wuchterl (1993) but using mixing length theory instead of zero entropy gradient convection. The curves are obtained for the following solar nebula densities: from bottom to top: 10^{-20}, 1.5×10^{-10} (MIZ_J_X, standard), 3×10^{-10}, 10^{-9}, 1.5×10^{-9} (JR15_XA), 2×10^{-9} , 2.5×10^{-9} (JR25_XA) , 3×10^{-9}[g/cm^3] (JR30_XA). Observe the convergence of the relations for the two smallest densities, that differ by more than ten orders of magnitude. Note the spreading of the envelope masses for convective outer envelopes (above MIZ_J_X) for nebula densities within a factor of 10 above the standard value.

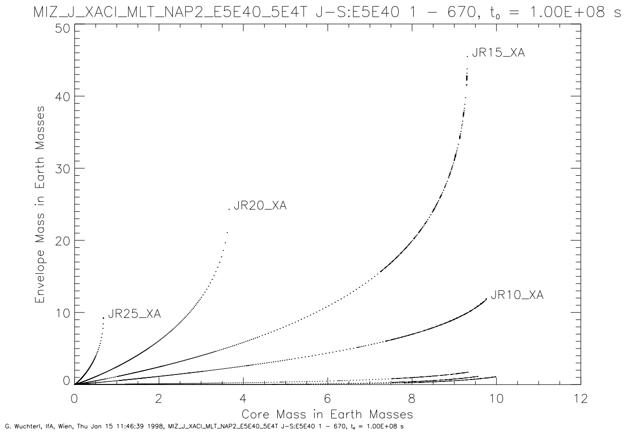

Figure 0.3: Masses of static protoplanetary envelopes as a function of core mass. Same as Fig. 0.2 but plotted linearly to emphasize the spreading in envelope mass for the convective outer envelopes occurring at larger nebula densities.

critical mass for various solar nebula conditions. For conditions resulting in a protoplanet with a (standard) radiative outer envelope, all evolutions are very similar: a fraction of the envelope is ejected at the critical mass. In all cases, after the end of the dynamical phase, core and envelope masses approach values similar to Uranus' and Neptune's mass. For nebula conditions resulting in convective outer envelopes, the accretion histories are qualitatively and quantitatively different: for midplane nebula densities that are enhanced by a factor of eight relative to minimum mass values, the envelopes continue to grow beyond a Jupiter mass.

In summary: the envelope masses of proto giant planets with a given core are independent of the conditions in the nebula and the position at which they occur as long as the outer parts of their envelopes are radiative. When convection sets in, the masses start to depend on the nebula density and the dynamical accretion behaviour changes in a way so that Jupiter-mass planets can form. 'Radiative' protoplanets ultimately result in Uranus/Neptune-mass planets, 'convective' protoplanets accrete to larger (Jupiter) masses.

0.4 Are any planetary masses favored?

In static and quasi-hydrostatic studies of giant planet formation the diversity of giant planet masses is usually attributed to processes that limit the supply of gas from the solar nebula. There are essentially two ways to limit the supply: (1) to remove the solar nebula and/or (2) to cut off the flow from the nebula onto the planetary envelope, e.g. by tidal truncation. Both of this mechanisms determining the masses of giant planets rely on the implicit assumption, that giant planets accrete gas essentially without intrinsic limits. The hydrodynamical studies outlined in the last two sections show, that this assumption is justified only for protoplanets with convective outer envelopes. By examining the conditions that possibly produce such envelopes we may discuss the question (3) of how frequently accretion of massive, i.e. Jupiter-like planets occurs.

Wuchterl (1993) has rewritten the Schwarzschild criterion to determine the nebula properties that result in convective proto giant planets at the critical mass. The resulting condition essentially determines a minimum nebula density for every temperature (for given constitutive relations and planetesimal accretion rates). Above that nebula density the outermost parts of a protoplanetary envelope become convective. Concerning the *envelope dynamics*, this is only an approximate condition (lower limit for the critical density), since the accretional behaviour changes only after a 'significant' fraction of the outer envelope has actually become convective (see Wuchterl 1993, 1995). However, even with the lower limit a considerable collection of solar nebula models (cf. Wuchterl 1993, Fig. 2) turns out to produce 'radiative' proto giant planets at the critical mass and, therefore, an accretional behaviour, that results in Uranus/Neptune-type planets. To find a convective proto giant planet, that accretes a Jupiter mass,the nebula midplane density had to be enhanced by a factor of 8 relative to minimum mass value.

In summary: Most solar nebula models in the literature produce 'radiative' proto giant planets. In all those nebulae Uranus/Neptune-mass objects should form since accretion to larger masses is hydrodynamically inhibited. Only in sufficiently dense nebulae producing 'convective' proto giant planets, the formation of Saturn and Jupiter is possible. Therefore, relying on the existing nebula models, hydrodynamic giant planet formation theories apparently favor Uranus/Neptune-mass planets. As a consequence, Jupiter-mass objects should be the exception.

0.5 Formation of giant planets at small orbital distances

Recently, a planetary mass companion of 51 Peg has been discovered at an orbital distance of 0.05 AU and 3 other giant planets close to stars are known presently (Mayor and Queloz 1995, and chapter by Marcy). Guillot et al. (1996) showed, that a giant planet can be stable over 51 Peg's age. Wuchterl (1996, 1997) addressed the question of forming a giant planet close to a star using standard nebula conditions at 0.05 AU and neglecting effects of possible limited gas supply.

The results of these radiation-hydrodynamical calculations, using a time dependent model for turbulent convection (Kuhfuß (1986),(1987)) to describe convective energy transfer are shown in Fig. 0.4 for two different assumptions about the nebula temperature (1250 and 600 K).

In both cases significant gas accretion sets in. At core masses of 7.5 $M_\oplus$ and 13.5 $M_\oplus$, respectively, envelope accretion takes over as the dominating mass accumulation process. The proto giant planets grow beyond a Jupiter mass despite the high nebula temperatures and the small Hill-sphere caused by the strong solar tidal forces. This is apparently overcompensated by the enhanced self-gravity of the envelope gas due to the fact, that the nebula gas-density at 0.052 AU is orders of magnitude higher than at Jupiter's position. When considering the high mass end of the protoplanetary evolutionary curves, above 100 $M_\oplus$ say, it has to be kept in mind, that gas-supply-limiting-processes, as tidal truncation or nebula dispersal have not been taken into account.

0.6 Conclusion

Hydrodynamical studies of planetary envelope accretion show, that Uranus/Neptune-type planets are the standard outcome of the giant planet formation process. Jupiters require nebula densities that are enhanced relative to present nebula models to hydrodynamically enable the onset of efficient envelope gas accretion.

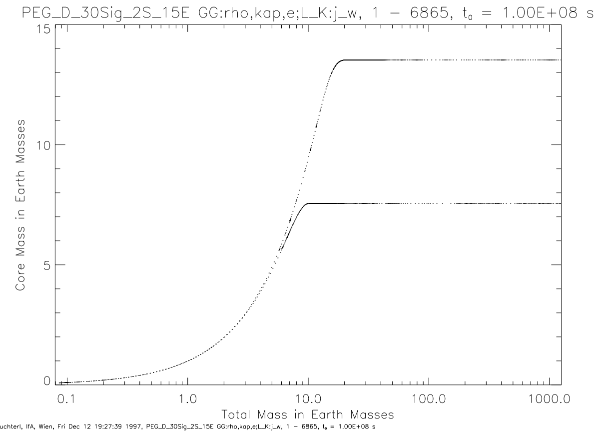

Figure 0.4: Formation of giant planets at 0.052 AU. Two radiation hydrodynamical calculations with time dependent convection are shown. See Wuchterl (1997) for details. At a few earth masses the curves separate and turn to the right as the envelopes begin to contribute a significant fraction to the total mass. At core masses of 13.5 $M_\oplus$ and 7.5 $M_\oplus$ for cases 1 (Standard) and 2 (reduced nebula temperature) respectively the curves turn sharply to the right as efficient gas accretion sets in and rapidly increases the envelope mass.

The formation of 51 Peg type giant planets at small orbital distances occurs if condensible element cores grow to a few earth masses in the presence of a standard solar nebula and a sufficient supply of nebula gas.

0.7 Acknowledgements

This work has been supported by the Österreichischer Fonds zur Förderung der wissenschaftlichen Forschung (FWF) under project numbers S-7305–AST, S-7307–AST.

Bibliography

[1] Alexander, D.R. and J.W. Ferguson (1994): Astrophys. Journ. **437**: 879

[2] Bodenheimer, P. and Pollack, J. B. 1986: Calculations of the Accretion and Evolution of Giant Planets: The Effects of Solid Cores. Icarus **67**, 391-408.

[3] Dorfi, E.A. and Feuchtinger, M.U. (1995): Adaptive Radiation Hydrodynamics of Pulsating Stars Comp. Phys. Comm. **89**: 69–90

[4] Götz, M. (1993): *Die Entwicklung von Proto-Gasplaneten mit Drehimpuls — Strahlungshydrodynamische Rechnungen*, Dissertation, Univ. Heidelberg.

[5] Guillot, T., Burrows, A., Hubbard, W.B., Lunine, J.I., Saumon,D. (1996): Giant planets at small orbital distances. Astrophys. Journ. **459**: L35–L38.

[6] Hayashi, C., K. Nakazawa and Y. Nakagawa (1985): Formation of the Solar System. In *Protostars and Planets II* (D.C.Black and M.S.Matthews, Eds.) Univ. of Arizona Press, Tucson.

[7] Kuhfuß, R. (1986): A model for time-dependent convection Astron. Astrophys. **160**, 116—120

[8] Kuhfuß, R. (1987): *Ein Modell für zeitabhängige, nichtlokale Konvektion in Sternen.* Ph. D. thesis, Tech. Univ. Munich.

[9] Lissauer, J.J. (1993): Planet Formation. Ann. Rev. Astron. Astrophys. **31**: 129–174

[10] Lissauer, J.J., Pollack, J.B., Wetherill, G.W. and Stevenson, D.J. (1995): Formation of the Neptune System. In *Neptune* (Univ. Arizona Press, Tucson).

[11] Mayor, M., Queloz, D. (1995): Nature **378**, 355.

[12] Mizuno, H. (1980): Formation of the Giant Planets. Prog. Theor. Phys. **64**, 544-557

[13] Pollack, J.B., O. Hubickyj, P. Bodenheimer, J.J. Lissauer, M. Podolak and Y. Greenzweig (1996): Formation of the Giant Planets by Concurrent Accretion of Solids and Gas. *Icarus*, **124**, 62–85.

[14] Stevenson, D.J. (1982): Formation of the Giant Planets. Planet. Space Sci. **30**, 755-764

[15] Tajima, N. and Y. Nakagawa (1995): *Giant Planet Formation: Dynamical Stability of the Envelope*, In *Comparative Planetology* (M.T. Chahine, M.F. A'Hearn, J.H. Rahe, Eds.),67–69 . Kluwer Acad. Publ., Dordrecht.

[16] Tajima, N. and Y. Nakagawa (1997): 'Evolution and Dynamical Stability of the Proto-giant-planet envelope', *Icarus* **126**, 282–292.

[17] Wuchterl, G. (1991): Hydrodynamics of Giant Planet Formation II: Model Equations and Critical Mass. Icarus, **91**, 39-52.

[18] Wuchterl, G. (1991): Hydrodynamics of Giant Planet Formation III: Jupiter's Nucleated Instability. Icarus, **91**, 53-64.

[19] Wuchterl, G. (1993): The Critical Mass for Protoplanets Revisited: Massive Envelopes Through Convection. Icarus **106**, 323–334.

[20] Wuchterl, G. (1995a): Giant Planet Formation. A comparative View of Gas-Accretion. In *Comparative Planetology* (M.T. Chahine, M.F. A'Hearn, J.H. Rahe, Eds.). (Kluwer Acad. Publ., Dordrecht) and
Earth, Moon, and Planets **67**, 51–65.

[21] Wuchterl, G. (1995b): Time dependent convection on self adaptive grids. Comp. Phys. Comm. **89**, 119–126.

[22] Wuchterl, G. (1996). 'Formation of Giant Planets Close to Stars'. 28th DPS-meeting, Tucson, AZ, 23.-26. Oct. 1996 (Poster). *Bull. Am. Astr. Soc.* **28,3** p. 1108 (Abstract).

[23] Wuchterl, G. (1997). 'Giant Planet Formation and the Masses of Extrasolar Planets'. In *Science with the VLT Interferometer* (F. Paresce, Ed.), 74–71. Springer, Berlin 1997.